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Gurukul for IITJEE Preparation
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MOCK TEST PAPER # 1

SOLUTION

IITJEE (Main) MATHEMATICS

1.(D) From diag. shaded region = $A-(B\cup C)$

2.(A) $u^2-2u+2=0$

$\alpha=1+i,\ \beta=(1-i)$,

And $x=\cot\theta-1$

$$\frac{(x+\alpha)^n-(x+\beta)^n}{(\alpha-\beta)}=\frac{[(\cot\theta-1)+(1+i)]^n-[(\cot\theta-1)+(1-i)]^n}{(1+i)-(1-i)}$$

$$\frac{\frac{(\cos\theta+i\sin\theta)^n}{\sin^n\theta}-\frac{(\cos\theta-i\sin\theta)^n}{\sin^n\theta}}{2i}=\frac{e^{in\theta}-e^{-in\theta}}{2i.\sin^n\theta}=\frac{\sin n\theta}{\sin^n\theta}$$

3.(C) $$\frac{\alpha^8(\alpha^2-1)-\beta^8(\beta^2-1)}{\alpha^{(\alpha+\beta+2)}-\beta^{(\alpha+\beta+2)}}=\frac{\alpha^8(\alpha^2-1)-\beta^8(\beta^2-1)}{(\alpha^9-\beta^9)}$$

$\because \alpha,\beta$ are roots of equation $x^2-7x-1=0$ i.e.; $\alpha^2-1=7\alpha$ and $\beta^2-1=7\beta$

$$\frac{\alpha^8(7\alpha)-\beta^8(7\beta)}{\alpha^9-\beta^9}=\frac{7(\alpha^9-\beta^9)}{\alpha^9-\beta^9}=7$$

4.(D) $$\prod_{r=1}^{50}\begin{bmatrix}1 & 2r-1\\0 & 1\end{bmatrix}=\underbrace{\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}}\begin{bmatrix}1 & 5\\0 & 1\end{bmatrix}\cdots\cdots\begin{bmatrix}1 & 99\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & (3+1)\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & (1+3)\\0 & 1\end{bmatrix}$$

$$\prod_{r=1}^{50}\begin{bmatrix}1 & 2r-1\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & \underbrace{1+3+5+\ldots\ldots 99}_{50\text{ terms}}\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 2500\\0 & 1\end{bmatrix}$$

5.(A) $$\Delta=\begin{vmatrix}x & x+a & x+2a\\x+1 & x+2a & x+4a\\x+2 & x+3a & x+6a\end{vmatrix}=\begin{vmatrix}x & x+a & x+2a\\1 & a & 2a\\1 & a & 2a\end{vmatrix}=0$$ $R_3\to R_3-R_2$

$R_2\to R_2-R_1$

6.(C) **Case I:** 4 boys and 2 girl

No. of ways of selection = ${}^6C_4\,{}^3C_2=15\times3=45$

Case II: 3 boys and 3 girls

No. of ways of selection = ${}^6C_3\,{}^3C_3=20\times1=20$

Total number of ways is 45 + 20 = 65

7.(C) $$f(n)=\sum_{r=1}^{n}r^2\left({}^nC_r\right)^2=\sum_{r=1}^{n}\left(r.{}^nC_r\right)^2=n^2\sum_{r=1}^{n}\left({}^{n-1}C_{r-1}\right)^2=n^2\sum_{r=1}^{n}{}^{n-1}C_{r-1}\,{}^{n-1}C_{n-r}$$

$$=n^2\ {}^{2n-2}C_{n-1}$$

$$f(5)=5^2\ {}^{2\times5-2}C_{5-1}=25\times{}^8C_4=1750$$

8.(D) $\sum_{r=1}^{n} a_r(1-a_r)+\sum_{r=1}^{n}\left(a_r^2+a^2-2aa_r\right)$

$$\sum_{r=1}^{n}\left(a^2-2aa_r+a_r\right)=na^2-2a(na)+na=na-na^2=na(1-a)=nab$$

9.(C) $1+\sum_{r=0}^{18}\{(r+1)(r+2)-(r+1)\}r!=k!$

$$1+\left[\sum_{r=0}^{18}(r+2)!-(r+1)!\right]=1+\left[(2!-1!)+(3!-2!)+\ldots\ldots(20!-19!)\right]=20!$$

$k!=20!$

$k=20$

k is not divisibly by 6

10.(C) $\int_0^x (x-u)f(u)du=e^{2x}-2x-1$

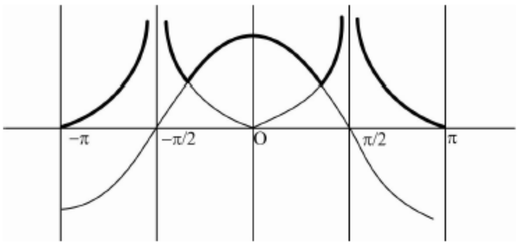
$$x\int_0^x f(u)du-\int_0^x uf(u)du=e^{2x}-2x-1$$

Diff. w.r.t. x : $\int_0^x f(u)du+x[f(x)]-xf(x)=2e^{2x}-2$

Again diff. w.r.t. x

$$f(x)=4e^{2x} \quad \Rightarrow \quad \lim_{x\to 0}\frac{4\left[e^{2x}-1\right]}{2x}\times 2=8$$

11.(A) Total four points where function is non differentiable.



12.(B) $f'(x)=3x^2+2ax+b+5\sin 2x\ge 0 \;\; \forall x\in R$

$3x^2+2ax+(b-5)\ge 0 \qquad \forall x\in R$

$D\le 0$

$(2a)^2-4\times 3(b-5)\le 0$

$a^2-3b+15\le 0$

13.(D) $f\left(\underbrace{x+\frac{y}{8}}_{u},\underbrace{x-\frac{y}{8}}_{v}\right)=xy$

$u=x+\frac{y}{8} \qquad \Rightarrow \qquad v=x-\frac{y}{8}$

On, adding and subtracting we get : $x=\frac{u+v}{2}$; $\quad y=4(u-v)$

$f(u.v)=2\left(u^2-v^2\right) \qquad \Rightarrow \qquad f(m,n)+f(n,m)=2\left(m^2-n^2\right)+2\left(n^2-m^2\right)=0$

$f(m,n)+f(n,m)=0$ For all value of m and n.

14.(C) Put $\left(\underbrace{\log\log.......x}_{8\text{ times}}\right)=t$

$$\Rightarrow \frac{1}{\left(\underbrace{\log\log.......x}_{7\text{ times}}\right)}\times\frac{1}{\left(\underbrace{\log\log.......x}_{6\text{ times}}\right)}..........\frac{1}{\log x}\times\frac{1}{x}dx=dt$$

$$\int\frac{dx}{t}=\log(t)+c=\underbrace{\log(\log\log.......x)}_{9\text{ times}}+c$$

15.(C) If $0\le x^2<1$ then $[x^2]=0$ and $1\le x^2<2$ then $[x^2]=1$

$$\Rightarrow \int_0^{\sqrt{2}}[x^2]dx=\int_0^1 0dx+\int_1^{\sqrt{2}}1dx=(\sqrt{2}-1)$$

16.(D) $|(\bar{a}\times\bar{b}).\bar{c}|=|\bar{a}||\bar{b}||\bar{c}|=\bar{a},\bar{b},\bar{c}$ are non-zero vectors

Possible only when $\sin\theta=1 \Rightarrow \cos\theta=0$ for any two vectors, where $\bar{a}.\bar{b}=\bar{b}.\bar{c}=\bar{c}.\bar{a}=0$

17.(A) Equation of line parallel to line $\frac{x}{2}=\frac{y}{3}=\frac{z-1}{-6}$ and through P(1, -2, 3) is $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{-6}=r$;

Any point of this line is $(2r+1, 3r-2, -6r+3)\equiv Q(\text{say})$

It is lies on the plane $x-y+z-5=0$

$$r=1/7;\ Q\equiv\left(\frac{9}{7},-\frac{11}{7},\frac{15}{7}\right)\Rightarrow PQ=1$$

18.(B) The point of intersection of $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}=\lambda$ and $\frac{x-4}{5}=\frac{y-1}{2}=\frac{z}{1}=\mu$ will be given by $2\lambda+1=5\mu+4, 3\lambda+2=2\mu+1, 4\lambda+3=\mu$

$\lambda=\mu=-1$ and then point is (-1, -1, -1)

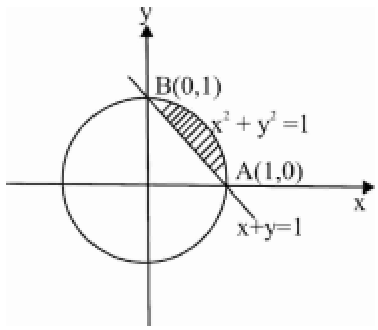
19.(B) $P(A)=P(B)=2P(C)$

Since these are mutually exclusive and exhaustive events,

$$P(A)=P(B)=2/5 \text{ and } P(C)=1/5$$

So, $P(B\cup C)=P(B)+P(C)=3/5$

20.(D) Given equation of circle and line are



$$x^2+y^2=1 ....(i)$$

and $x+y=1 ....(ii)$

From equation (i) and (ii),

$$x^2+(1-x)^2=1$$

$$\Rightarrow x^2+1+x^2-2x=1$$

$$2x^2-2x=0\Rightarrow 2x(x-1)=0$$

$$x=0, x=1\Rightarrow y=1, y=0$$

Point of intersection of circle and line are A(1,0), B(0,1)

Required Area $=\int_0^1\left[\sqrt{1-x^2}-(1-x)\right]dx=\left[\frac{x\sqrt{1-x^2}}{2}+\frac{1}{2}\sin^{-1}x-x+\frac{x^2}{2}\right]_0^1=\frac{1}{2}\frac{\pi}{2}-1+\frac{1}{2}=\left(\frac{\pi}{4}-\frac{1}{2}\right)$ sq unit

21.(A) We have $\frac{dy}{dx}+\frac{2yx}{(1+x^2)}=\frac{1}{(1+x^2)^2}$

Comparing the given differential equation with linear differential equation

$$\frac{dy}{dx}+Py=Q$$

We get $P=\frac{2x}{1+x^2}$ and $Q=\frac{1}{(1+x^2)^2}$

Now, If $e^{\int pdx}=e^{\int\frac{2x}{1+x^2}dx}=e^{\log(1+x^2)}=(1+x^2)$

Solution of differential equation is

$$y(1+x^2)=\int\frac{1}{(1+x^2)^2}dx+c$$

$$\Rightarrow \quad y(1+x^2)=\int\frac{1}{(1+x^2)}dx+c$$

$$y(1+x^2)=\tan^{-1}x+c$$

22.(B) Let the equation of circles be

$$S_1=x^2+y^2-3x-4y+5=0\ ;$$

$$S_2=x^2+y^2-\frac{7}{3}x+\frac{8}{3}y+\frac{11}{3}=0.$$

The equation of common chord is $S_1-S_2=0$

$$-2x-20y+4=0$$

$$y=-\frac{x}{10}+\frac{1}{5}$$

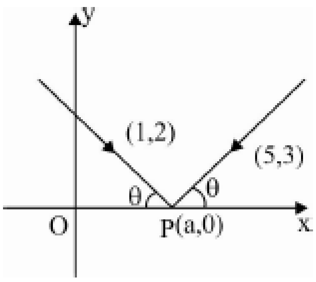
Gradient of radical axis $=-1/10$

23.(C) Slope of incident ray is $\frac{0-2}{a-1}=\tan(\pi-\theta)$

$$\tan\theta=\frac{2}{a-1}\ \textbf{......(i)}$$

Slope of reflected ray is $\frac{3-0}{5-a}=\tan\theta$ .....(ii)

From equation (i) and (ii) $\frac{2}{a-1}=\frac{3}{5-a}\Rightarrow a=\frac{13}{5}$



24.(A) Given equation of ellipse is $\frac{x^2}{16}+\frac{y^2}{9}=1$

Here $a^2=16$ and $b^2=9$,

$$b^2=a^2(1-e^2)\Rightarrow 9=16(1-e^2)$$

$$e=\frac{\sqrt{7}}{4}$$

Thus, the foci are $(\pm\sqrt{7},0)$

The radius of the required circle $=\sqrt{(\sqrt{7}-0)^2+3^2}=\sqrt{7+9}=4$

25.(B) Let any point on the line segment PQ is $R(\alpha,\beta)$,

then $\alpha=\frac{\lambda(1)+1}{\lambda+1}=1$, and $\beta=\frac{3\lambda+1}{\lambda+1}$

A point is inside parabola $y^2=4x$, , if $y^2-4x<0$

$$\left(\frac{3\lambda+1}{\lambda+1}\right)^2-4(1)<0$$

$$\Rightarrow \quad \left(\frac{3\lambda+1}{\lambda+1}+2\right)\left(\frac{3\lambda+1}{\lambda+1}-2\right)<0 \quad \Rightarrow \quad (5\lambda+3)(\lambda-1)<0$$

$$\Rightarrow \quad -\frac{3}{5}<\lambda<1$$

26.(D) $\tan^{-1}\left(\frac{\tan x}{4}\right)+\tan^{-1}\left(\frac{3\sin 2x}{5+3\cos 2x}\right)$

$$=\tan^{-1}\left(\frac{\tan x}{4}\right)+\tan^{-1}\left(\frac{\frac{6\tan x}{1+\tan^2 x}}{5+\frac{3\left(1-\tan^2 x\right)}{1+\tan^2 x}}\right)=\tan^{-1}\left(\frac{\tan x}{4}\right)+\tan^{-1}\left(\frac{6\tan x}{8+2\tan^2 x}\right)$$

$$=\tan^{-1}\left(\frac{\tan x}{4}\right)+\tan^{-1}\left(\frac{3\tan x}{4+\tan^2 x}\right)=\tan^{-1}\left(\frac{\frac{\tan x}{4}+\frac{3\tan x}{4+\tan^2 x}}{1-\frac{3\tan^2 x}{4\left(4+\tan^2 x\right)}}\right)$$

$$=\tan^{-1}\left(\frac{16\tan x+\tan^3 x}{16+\tan^2 x}\right)=\tan^{-1}(\tan x)=x$$

27.(A) Solving 3x + 4y = 9 and y = mx + 1, we get $x=\frac{5}{3+4m}$

x is an integer

3 + 4m = 1,-1, 5, -5

$m=-\frac{2}{4},-\frac{4}{4},\frac{2}{4},\frac{-8}{4}$. So, m has two integer values.

28.(C) The sum of items $49\times100-(40+20+50)+(60+70+80)=5000$

Means of 100 items = 5000/100 = 50

29.(B) Given that, $\sin\theta=-\frac{4}{5}$ and θ lies in the third quadrant.

$$\cos\theta=-\sqrt{1-\frac{16}{25}}=-\frac{3}{5}$$

$$\cos\theta/2=\pm\sqrt{\frac{1+\cos\theta}{2}}=\sqrt{\frac{1-\frac{3}{5}}{2}}=\pm\sqrt{\frac{1}{5}}$$

But we take $\cos\frac{\theta}{2}=-\frac{1}{\sqrt{5}}$. Since, if θ lies in the third quadrant, then $\theta/2$ will be in second quadrant.

Hence, $\cos\theta/2=-\frac{1}{\sqrt{5}}$

30.(A)

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